AP Calculus Summer Packet
Answer Key

Reminders:

1. This is **not** an assignment.
2. This will **not** be collected.
3. You **WILL** be assessed on these skills at various times throughout the course.
4. You are expected to be able to do all the questions in this packet **without** a calculator except for those in Section XXI.

I) ***Simplifying fractions:***

1) $\frac{1}{x}+\frac{1}{y}=\frac{y+x}{xy}$

2) $\frac{1}{x}+\frac{1}{x^2}=\frac{x+1}{x^2}$

3) $\frac{\frac{1}{x}+1}{\frac{1}{x}}=1+x$

4) $\frac{\frac{x}{x+y}}{x}=\frac{1}{x+y}$

5) $\frac{\frac{1}{x+h}+\frac{1}{x}}{x}=\frac{2x+h}{x^2(x+h)}$

II) ***Factoring:***

6) x^2-16 $(x+4)(x-4)$

7) x^2-x-6 $(x-3)(x+2)$

8) $6x^2-x-2$ $(2x+1)(3x-2)$

9) $4x^3-19x^2-5x$ $x(x-5)(4x+1)$

10) x^2+9 Prime (using $\mathbb{R}$ only)

11) x^4-13x^2-30 $(x^2-15)(x^2+2)$

12) x^3+27 $(x+3)(x^2-3x+9)$

13) x^3-8 $(x-2)(x^2+2x+4)$

14) $(2x-3)^3(x+1)+(x-3)(2x-3)^2$ $2(x^2-3)(2x-3)^2$

15) $(3x-2)^{-4}(x+3)+(x+3)^2(3x-2)^{-3}$ $\frac{(x+3)(3x^2+7x-5)}{(3x-2)^4}$

III) ***Solving:***

16) $x^2+5x-24=0$ $x=-8,3$

17) $x^2-9=5$ $x=\pm\sqrt{14}$

18) $3x^2-5x-2=0$ $x=-\frac{1}{3},2$

19) $x^2-4x=0$ $x=0,4$

20) $(x-1)(x^2-11x+30)=0$ $x=1,5,6$

21) $\sqrt{x}+1=41$ $x=1600$

22) $\frac{y}{x+1}=\frac{z}{x}$ $x=\frac{z}{y-z}$

23) $\sqrt[3]{x+1}-4=-1$ $x=26$

24) $x^{-2}=\frac{1}{9}$ $x=\pm3$

25) $2\sqrt{x}=x-3$ $x=9$

(x = 1 is extraneous.)

26) $\frac{8+x}{x}-5=0$ $x=2$

27) $x^{-1}=-3$ $x=-\frac{1}{3}$

28) $x^{\frac{4}{3}}=81$ $x=27$

29) $3x^2-6x-24\le0$ $[-2,4]$

30) $\frac{2x-1}{(x+2)(x^2+3)}=0$ $x=\frac{1}{2}$

31) $x^3-2x^2-5x+10=0$ $x=2,\pm\sqrt{5}$

IV) ***Are the following expressions equal to*** $\ln 4$ **?**

32) $2\ln 2$ **yes**

33) $\dfrac{\ln 8}{\ln 2}$ **no**

34) $\ln 8 - \ln 2$ **yes**

35) $\ln 4 + \ln 1$ **yes**

36) $\ln 4 \bullet \ln 1$ **no**

36) $(\ln 2)^2$ **no**

V) *Equations of lines:*

37) Find the equation of the line that has a slope of 5 and passes through the point (3, -4). $y+4=5(x-3)$ or $y=5x-19$

38) Find the equation of the line that passes through the points (4, 1) and (3, -2).

$y-1=3(x-4)$ or $y+2=3(x-3)$ or $y=3x-11$

39) Find the equation of the line that passes through the points (-2, 1) and is parallel to the line $4x+2y=-1$**.** $y-1=-2(x+2)$ or $y=-2x-3$

40) Find the equation of the line that has a slope of 0 and passes through the point (-5, 1). $y=1$

41) Find the equation of the line that passes through the origin and is perpendicular to the line $3x+4y=-7$**.** $y=\dfrac{4}{3}x$

42) Find the equation of the line that has an undefined slope and passes through the point (4, -5). $x=4$

VI) ***Intercepts:***

43) Find the *x* and *y* intercepts of $x^2 + y^2 = 9$**.**

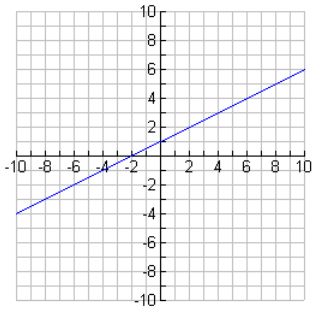
x-intercepts: (3, 0) and (-3, 0). *y*-intercepts: (0, 3) and (0, -3)

44) Find the equation of the line that has an *x*-intercept of 5 and a *y*-intercept of 3.

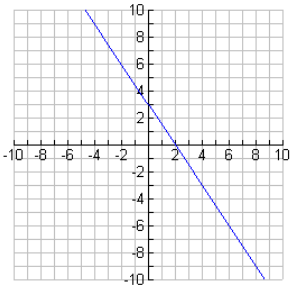
$y - 3 = -\frac{3}{5}(x - 0)$ or $y - 0 = -\frac{3}{5}(x - 5)$ or $y = -\frac{3}{5}x + 3$

VII) ***Equations of graphs:***

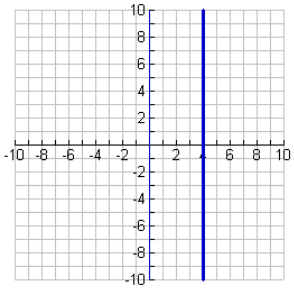
45) $y = \frac{1}{2}x + 1$



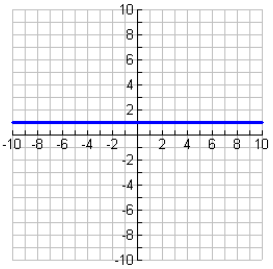
46) $y = -\frac{3}{2}x + 3$



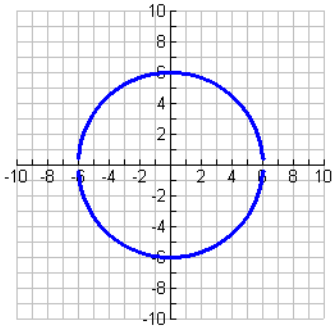
47) $x = 4$



48) $y = 1$



49) $x^2 + y^2 = 36$



VIII) ***Given the slope, sketch the following lines:***

[If you do not trust your sketches, see your instructor.]

50) Sketch a line with a slope of 2.

51) Sketch a line with a slope of $\frac{1}{2}$.

52) Sketch a line with a slope of -2.

53) Sketch a line with a slope of $-\frac{1}{2}$.

IX) ***Sketch the following graphs:*** **[You can check your answers with a graphing calculator, but please know that you are expected to know what these look like without the use of a graphing calculator.]**

54) $f(x)=3x+1$

55) $f(x)=x^2$

56) $f(x)=|x|$

57) $f(x)=x^3$

58) $x=3$

59) $y=-4$

60) $f(x)=\ln x$

61) $f(x)=\sqrt{x}$

62) $f(x)=\frac{1}{x}$

63) $f(x)=\frac{1}{x^2}$

64) $f(x)=|x+1|$

65) $f(x)=x^2+2x-3$

66) $f(x)=x^3+1$

67) $f(x)=(x+1)^2$

68) $f(x)=-x^2+1$

69) $f(x)=(x+1)^{1/3}$

70) $f(x)=x^{2/3}$

71) $f(x)=e^x$

72) $f(x)=-\frac{1}{x}$

73) $x^2+y^2=25$

X) ***Absolute value:***

74) $|x|=\begin{cases} x, & x\geq 0 \\ -x, & x<0 \end{cases}$

75) $|x-1|=\begin{cases} x-1, & x\geq 1 \\ -x+1, & x<1 \end{cases}$

XI) ***Domain and range:***

76) $f(x)=x-1$ $D:\mathbb{R}, R:\mathbb{R}$

77) $f(x)=\dfrac{1}{x}$ $D:\mathbb{R}\neq 0, R:\mathbb{R}\neq 0$

78) $f(x)=\dfrac{1}{x^2+1}$ $D:\mathbb{R}, R:0<y\leq 1$

79) $f(x)=e^x$ $D:\mathbb{R}, R:\mathbb{R}>0$

80) $f(x)=\sqrt{x-4}$ $D:\mathbb{R}\geq 4, R:\mathbb{R}\geq 0$

81) $f(x)=|x-1|+2$ $D:\mathbb{R}, R:\mathbb{R}>2$

82) $f(x)=\ln x$ $D:\mathbb{R}>0, R:\mathbb{R}$

83) $f(x)=\sqrt{x^2-3x-4}$ $D:(-\infty,-1),(4,\infty), R:\mathbb{R}\geq 0$

84) $f(x)=\dfrac{1}{x+6}-10$ $D:\mathbb{R}\neq -6, R:\mathbb{R}\neq -10$

XII) ***Inverses:***

85) $f(x)=x+3$ $f^{-1}(x)=x-3$

86) $f(x)=\sqrt{x}$ $f^{-1}(x)=x^2$

87) $f(x)=\dfrac{x}{x+2}$ $f^{-1}(x)=\dfrac{-2x}{x-1}$

88) $f(x)=\ln x$ $f^{-1}(x)=e^x$

XIII) ***Compositions of functions if:***

$f(x)=x^3+1$, $g(x)=x^2-2$, **and** $j(x)=x+3$

89) $f(2)=9$

90) $f\left(j(x)\right)=(x+3)^3+1$

91) $f\left(j(2)\right)=126$

92) $g\left(g(x)\right)=x^4-4x^2+2$

93) $f(x+h)=x^3+3x^2h+3xh^2+h^3+1$

94) $\dfrac{f(x+h)-f(x)}{h}=3x^2+3xh+h^2$

XIV) ***Simultaneous equations:***

95a) $\begin{aligned} 2x+3y&=8 \\ x+2y&=5 \end{aligned}$

(1, 2)

95b) $\begin{aligned} y&=x^2+2x+9 \\ 7x+y&=19 \end{aligned}$

(-10, 89) and (1, 12)

96a) The length, *l*, of a certain rectangle is twice the width, *w*. Write an equation for the perimeter the rectangle as a function of the width, *w*. ***P=6w***

96b) If the area of the rectangle described above is 50 square feet, find the length and the width of the rectangle. ***W = 5 feet and l = 10 feet***

XV) ***Intersection of curves:***

97) Find the point of intersection between the lines $y=x+1$ **and** $3y-x=5.$

(1, 2)

98) Find the point of intersection between the lines $y=x+7$ **and the curve** $y=x^2+2x+5$**. Also sketch the area between the graphs.**

(-2, 5) and (1, 8)

XVI) ***What do the following mean?*** [If you do not trust your answers, see your instructor.]

99) a graph is in the first quadrant

100) $f(2)=5$

101) an expression is a function

102) a zero of a function is 4

103) ***y*** **is directly proportional to** ***x*** **(give an example)**

104) the coefficient of the third term is 5 (give an example)

105) a function has only one root

106) a function is a polynomial

107) two triangles are similar

108) a function is even

109) a function is odd

XVII) ***What are the following formulas?***

110) Quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

111) Pythagorean Theorem $a^2 + b^2 = c^2$

112) the hypotenuse of a 45-45-90 isosceles right triangle with a leg of length *x*. $x\sqrt{2}$

113) the hypotenuse of a 30-60-90 right triangle with shortest leg having a length of *x*. $2x$

114) the volume of a sphere $V = \dfrac{4}{3}\pi r^3$

115) the volume of a cylinder $V = \pi r^2 h$

116) the volume of a cone $V = \dfrac{1}{3}\pi r^2 h$

117) the volume of a box with a square base $V = x^2 h$

118) the surface area of a sphere $SA = 4\pi r^2$

119) the surface area of a cylinder with no top $SA = \pi r^2 + 2\pi rh$

120) the area of a triangle $A = \dfrac{1}{2}bh$

121) the area of a trapezoid $A = \dfrac{1}{2}h(b_1 + b_2)$

122) the cross section through the center of a sphere $A = \pi r^2$

123) the volume of a prism that has an equilateral triangle with side of length *x* and height of length *y* [Ambiguous question -- don't worry about this one.]

124) area of an equilateral triangle in terms of the length of a side *s* $A = \dfrac{s^2\sqrt{3}}{4}$

XVIII) ***Solve using similar triangles:***

125) A six foot man is standing 10 feet away from a 20 foot lamppost. What is the length of his shadow? **30/7 feet**

126) Water is dripping out of a conical figure that has a diameter of 8 inches and a height of 12 inches. When the depth of the water is 8 inches, what is the radius of the water? **8/3 inches**

XIX) ***Find the equations of the horizontal and vertical asymptotes of each function:***

127) $y = \dfrac{1}{x-1}$

$H.A.: y = 0,\ V.A.: x = 1$

128) $y = \dfrac{x^3}{x^3 - 1}$

$H.A.: y = 1,\ V.A.: x = 1$

XX) ***Exponent Rules: Which of the following are true?***

129) $x^0 = 1$ **True**

130) $x^{-2} = \frac{1}{x^2}$ **True**

130) $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$ **NO !!!**

131) $x^5 \bullet x^3 = x^{15}$ **No**

132) $x^5 \bullet y^5 = (xy)^5$ **True**

133) $(x^3)^5 = x^8$ **No**

134) $x^{5-w} = \frac{x^5}{x^w}$ **True**

135) $x^{t+5} = (x^t)^5$ **No**

136) $\sqrt{\frac{9}{4}} = \frac{3}{2}$ **True**

137) $(4x)^{\frac{1}{2}} = 2x$ **No**

138) $\sqrt{\frac{1}{x}} = x^{-\frac{1}{2}}$ **True**

138) $\sqrt{x^2} = x$ **Technically, this is false.** $\sqrt{x^2} = |x|$ **because we do not know if *x* is positive or negative. Just be careful.**

139) $\sqrt{x^2 - 25} = x - 5$ **NO!!!**

140) $x^{\frac{4}{3}} = \sqrt[4]{x^3}$ **No**

141) $\left(x^{\frac{1}{2}} + y^{\frac{1}{2}}\right)^2 = x + y$ **NO!!!**

142) $x^{\frac{-2}{3}} = \frac{1}{\sqrt[3]{x^2}}$ **True**

143) $e^{\ln x^2} = x^2$ **True**

143) $\ln e^3 = 3$ **True**

144) $e^{2\ln 2 - \ln 5} = \frac{4}{5}$ **True**

145) $\ln x^2 = (\ln x)^2$ **No**

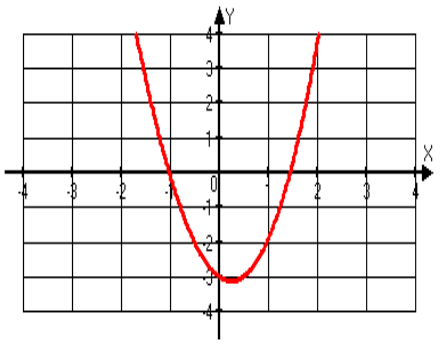
146) Expand using the properties of logarithms: $\ln \sqrt[3]{\frac{(3x+7)^4 (x+10)^3}{(5x-8)^2}}$

$$\frac{1}{3}\left[4\ln(3x+7) + 3\ln(x+10) - 2\ln(5x-8)\right]$$

147) Condense into a single logarithmic expression using the properties of logarithms: $17\ln x - \frac{2}{3}\ln(x^5+5)$

$$\ln\left(\frac{x^{17}}{\sqrt[3]{(x^5+5)^2}}\right)$$

XXI) ***Using the graphing calculator:***



148) Graph $y = 0.1x^3 + 2x^2 - x - 3$ on the *x*-*y* plane on the right:

149) Find the roots of the equation above.

-20.418 (not shown in the graph -- danger of graphing calculator), -1.021, 1.439

150) Find the point of intersection for the graphs $y = x^3 + x - 3$ and $y = 2x + 4$.

(2.087, 8.173)

151) Find the maximum value for the graph $f(x) = -x^4 + x - 4$.

-3.528

152) For the function in #151, find the intervals on which $f(x)$ is increasing.

(- ∞ , 0.630)

XXII) ***What are the following trigonometric identities?***

153) $\sec x = \dfrac{1}{\cos x}$

154) $\csc x = \dfrac{1}{\sin x}$

155) $\tan x = \dfrac{\sin x}{\cos x}$

156) $\cot x = \dfrac{\cos x}{\sin x}$

157) $\cos^2 x - 1 = -\sin^2 x$

158) $\sec^2 x - 1 = \tan^2 x$

159) $\cot^2 x + 1 = \csc^2 x$

XXIII) ***Evaluate the following expressions:***

160) $\sin\left(\dfrac{\pi}{6}\right)$ $\dfrac{1}{2}$

161) $\cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$ $\dfrac{\pi}{6}$

162) $\tan\left(\dfrac{7\pi}{6}\right)$ $\dfrac{\sqrt{3}}{3}$

163) $\cos(0)$ 1

164) $\cos\left(\dfrac{\pi}{4}\right)$ $\dfrac{\sqrt{2}}{2}$

165) $\csc\left(\dfrac{-5\pi}{6}\right)$ -2

166) $\sec(\pi)$ -1

167) $\cot\left(\frac{-\pi}{2}\right)$ 0

168) $\sin^{-1}\left(\frac{1}{2}\right)$ $\frac{\pi}{6}$

169) $\tan\left(\frac{\pi}{2}\right)$ **undefined**

170) $\sin^2\left(\frac{5\pi}{6}\right)$ $\frac{1}{4}$

171) $\cot\left(\frac{2\pi}{3}\right)$ $-\frac{\sqrt{3}}{3}$

172) $\sin\left(\frac{\pi}{2}\right)$ 1

173) $\cot^{-1}(-1)$ $\frac{3\pi}{4}$

174) $\sec\left(\frac{3\pi}{4}\right)$ $-\sqrt{2}$

175) $\tan^{-1}(-1)$ $-\frac{\pi}{4}$

176) $\csc(\pi)$ undefined

177) $\sec^2\left(\frac{\pi}{4}\right)$ 2

XXIV) ***Sketch one period of the following trigonometric graphs:***
[Check with your instructor if you do not know how to graph these by hand.]

178) $y = \sin x$

179) $y = \cos x$

180) $y = \tan x$

181) $y = \sec x$

182) $y = \csc x$

183) $y = \cot x$

XXV) ***Solve the following trigonometric equations for the given domain:***

184) $\sin x = \cos x$ on $[0, 2\pi]$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$